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Research Article

Magnetized Bulk Viscous Bianchi Type-IX String Cosmological Model with Cosmological Term Λ

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Abstract: We have investigated Bianchi type-IX string cosmological model with cosmological term Λ in the presence of bulk viscous fluid with electromagnetic field. To obtain the deterministic solution we assume that bulk viscosity ξ is proportional to the expansion θ; proper energy density ρ is proportional to the string tension density λ and Λ is proportional to R^{-3}, where R is scale factor. Various physical and geometrical aspects of the model are also discussed.

Keywords: Bianchi type-IX, viscous fluid, electromagnetic field, cosmological term.

1. INTRODUCTION

In recent years, Bianchi type-IX cosmological models play an important role for relativistic studies as these models permits not only expansion but also shear and rotation and in general, these models are anisotropic. Many researchers have taken keen interest to study these models because well-known solutions like Robertson Walker space-time, the De-Sitter space-time, the Taub-Nut space-time etc. are specific case of Bianchi type-IX universe. Bianchi type-IX cosmological models in different context have been studied by number of authors' viz. Parikh et al.[8], Bali and Yadav[2], Bali and Dave[1], Tiwari et al.[14], Tyagi and Sharma[17].

Additionally the magnetic field has the significant role at the cosmological scale and is present in galactic and intergalactic space. It plays an important role in the explanation of the energy distribution in the universe as it contains extremely ionized matter. Different cosmological models in different Bianchi types with electromagnetic field are considered by various researchers namely Tyagi et al.[6, 15, 18], Mete et al.[7], Patil and Bhojne[9], Wang[19], Singh *et al*.[10], Humad et al.[5], Deo *et al*.[3], Singh, Tyagi and Tripathi[11].

Dubey *et al*.[4] have studied Bianchi type-I viscous fluid cosmological models with cosmological term $\Lambda(t)$. Tiwari *et al.*[12] have investigated Bianchi type-V cosmological models with viscous fluid and varyingΛ. LRS Bianchi type-II homogeneous cosmological model for perfect fluid with electromagnetic field and variable Λ has been studied by Tyagi *et al.*[16]. Bianchi type-I string cosmological model with bulk viscosity and time-dependent Λ term has been investigated by Tiwari and Sharma[13].

In this paper, we study some homogeneous Bianchi type-IX string cosmological model with cosmological term Λ in the presence of bulk viscous fluid with electromagnetic field. To obtain the deterministic solution we assume that bulk viscosity ξ is proportional to the expansion θ; proper energy density ρ is proportional to the string tension density λ and Λ is proportional to R^{-3}, where R is scale factor. Various physical and geometrical features of the model are also discussed.

2. METRIC AND FIELD EQUATIONS

We consider Bianchi type-IX metric of the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + (B^2sin^2y + A^2cos^2y)dz^2 - 2A^2cosy\,dx\,dz \quad (1)$$

Where A and B are functions of t alone.

The energy momentum tensor for a cloud of strings with viscous fluid distribution and magnetic field is given by

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \xi\theta\left(v_i v^j + g_i^j\right) + E_i^j \quad (2)$$

with $v_i v^i = -x_i x^i = -1$ and $v^i x_i = 0$ (3)

Here ρ is proper energy density, λ is string tension density, x^i is the unit space like vector specifying the direction of strings and v^i is the unit time like vector.

In a co-moving coordinate system, we have

$$v^i = (0,0,0,1);\ x^i = \left(\frac{1}{A}, 0,0,0\right) \quad (4)$$

If ρ_p is the particle density of configuration, then

$$\rho = \rho_p + \lambda \quad (5)$$

The electromagnetic field E_i^j is defined as

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$$E_i^j = \frac{1}{4\pi}\left[-F_{il}F^{jl} + \frac{1}{4}g_i^j F_{lm}F^{lm}\right] \quad (6)$$

Maxwell's equation is

$$\frac{\partial}{\partial x^j}\left(F^{ij}\sqrt{-g}\right) = 0 \quad (7)$$

As, the incident magnetic field is taken along x-axis, therefore with the help of Maxwell's equation (7), the only non-vanishing component of F_{ij} is

$$F_{23} = H\sin y \ \ (constant) \quad (8)$$

The components of electromagnetic field E_i^j are given by

$$E_1^1 = \frac{H^2}{8\pi B^4} = -E_2^2 = -E_3^3 = E_4^4 \quad (9)$$

The Einstein's field equation (in the gravitational unit c=8πG=1) is given by

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda g_i^j = -T_i^j \quad (10)$$

where R_i^j is Ricci tensor, R=$g^{ij}R_{ij}$ is Ricci scalar.

The Einstein's field equation (10) for metric (1) leads to

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{1}{B^2} - \frac{3A^2}{4B^4} + \Lambda = \lambda + \xi\theta - \frac{H^2}{8\pi B^4} \quad (11)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \frac{A^2}{4B^4} + \Lambda = \xi\theta + \frac{H^2}{8\pi B^4} \quad (12)$$

$$\frac{2A_4B_4}{AB} + \frac{B_4^2}{B^2} + \frac{1}{B^2} - \frac{A^2}{4B^4} + \Lambda = \rho - \frac{H^2}{8\pi B^4} \quad (13)$$

3. SOLUTION OF FIELD EQUATIONS

The Field Equations (11)-(13) are system of three equations with seven unknown parameters A, B, Λ, λ, ρ, ξ, θ. Thus initially the system is undetermined, so we need four extra conditions. To obtain the deterministic solution, we assume the following conditions:

Bulk viscosity ξ is proportional to the expansion θ

i.e. $\xi = k_1\theta$ (14)

Proper energy density ρ is proportional to the string tension density λ

i.e. $\rho = l\lambda$ (15)

Expansion θ is proportional to the shear σ, which leads to

$$A = B^n \quad (16)$$

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and Λ is proportional to R^{-3}

$$\text{i.e.}\quad \Lambda = \frac{\alpha}{R^3} = \frac{\alpha}{AB^2} \tag{17}$$

where k_1, l, α are constants of proportionality.

Using condition (15), (16) and (17) in field equations (11) and (13), we get

$$\frac{(1-l)B_4^2}{B^2} - \frac{2lB_{44}}{B} + \frac{2nB_4{}^2}{B^2} + \frac{1-l}{B^2} + \frac{3l-1}{4B^{4-2n}} + \frac{(1-l)\alpha}{B^{n+2}} + \xi\theta l + \frac{(1-l)H^2}{8\pi B^4} = 0 \tag{18}$$

Particular for this model, mathematically θ is given by

$$\theta = \frac{(n+2)B_4}{B} \tag{19}$$

Hence by equation (14)

$$\xi\theta = \frac{k_1(n+2)^2 B_4{}^2}{B^2} \quad , where\ n \neq -2 \tag{20}$$

Using equation (20) in equation (18), we get

$$B_{44} + \frac{\beta B_4^2}{2B} = \frac{\gamma}{2B} + \frac{\delta}{2B^{3-2n}} + \frac{\gamma\alpha}{2B^{n+1}} + \frac{k\gamma}{2B^3} \tag{21}$$

where

$$\frac{l-1-2n-lk_1(n+2)^2}{l} = \beta; \quad \frac{1-l}{l} = \gamma; \quad \frac{3l-1}{4l} = \delta; \quad \frac{H^2}{8\pi} = k$$

Now, let $B_4 = f(B)$ then equation (21) reduces to the form

$$\frac{df^2}{dB} + \frac{\beta f^2}{B} = \frac{\gamma}{B} + \frac{\delta}{B^{3-2n}} + \frac{\gamma\alpha}{B^{n+1}} + \frac{k\gamma}{B^3} \tag{22}$$

On integrating equation (22), we obtain

$$f^2 = \frac{\gamma}{\beta} + \frac{\delta}{(\beta+2n-2)B^{2(1-n)}} + \frac{\gamma\alpha}{(\beta-n)B^n} + \frac{k\gamma}{(\beta-2)B^2} + \frac{M}{B^\beta} \tag{23}$$

where M is constant of integration.

From equation (23), we get

$$\int \frac{dB}{\sqrt{\frac{\gamma}{\beta} + \frac{\delta}{(\beta+2n-2)B^{2(1-n)}} + \frac{\gamma\alpha}{(\beta-n)B^n} + \frac{k\gamma}{(\beta-2)B^2} + \frac{M}{B^\beta}}} = t + N \tag{24}$$

where N is integrating constant.

Value of B can be determined by equation (24).

Hence, by suitable transformation of co-ordinates, metric (1) reduces to

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$$ds^2 = -\frac{dT^2}{\left[\frac{\gamma}{\beta} + \frac{\delta}{(\beta+2n-2)T^{2(1-n)}} + \frac{\gamma\alpha}{(\beta-n)T^n} + \frac{k\gamma}{(\beta-2)T^2} + \frac{M}{T^\beta}\right]} + T^{2n}dX^2 + T^2dY^2 + (T^2 sin^2Y + T^{2n}cos^2Y)dZ^2 - 2T^{2n}cosY\, dX\, dZ \quad (25)$$

where B=T, x=X, y=Y and z=Z.

4. PHYSICAL AND GEOMETRICAL ASPECTS

For the model (25), energy density (ρ), string tension density (λ), particle energy density (ρ_p), expansion(θ), bulk viscosity (ξ), shear (σ) and cosmological term Λ are given by

$$\rho = \left[\frac{\beta+\gamma(2n+1)}{\beta}\right]\frac{1}{T^2} + \left[\frac{4\delta(2n+1)-(\beta+2n-2)}{4(\beta+2n-2)}\right]\frac{1}{T^{4-2n}} + \left[\frac{\beta-n+\gamma(2n+1)}{\beta-n}\right]\frac{\alpha}{T^{n+2}} + \left[\frac{k\{(\beta-2)+\gamma(2n+1)\}}{(\beta-2)}\right]\frac{1}{T^4} + \frac{(2n+1)M}{T^{\beta+2}} \quad (26)$$

$$\lambda = \left[\frac{\beta+\gamma(2n+1)}{\beta l}\right]\frac{1}{T^2} + \left[\frac{4\delta(2n+1)-(\beta+2n-2)}{4l(\beta+2n-2)}\right]\frac{1}{T^{4-2n}} + \left[\frac{\beta-n+\gamma(2n+1)}{(\beta-n)l}\right]\frac{\alpha}{T^{n+2}} + \left[\frac{k\{(\beta-2)+\gamma(2n+1)\}}{(\beta-2)l}\right]\frac{1}{T^4} + \frac{(2n+1)M}{lT^{\beta+2}} \quad (27)$$

$$\rho_p = \frac{(l-1)\rho}{l} \quad (28)$$

Where $l \neq 1$ and ρ is given by equation (26).

$$\theta = (n+2)\left[\frac{\gamma}{\beta T^2} + \frac{\delta}{(\beta+2n-2)T^{4-2n}} + \frac{\gamma\alpha}{(\beta-n)T^{n+2}} + \frac{k\gamma}{(\beta-2)T^4} + \frac{M}{T^{\beta+2}}\right]^{1/2} \quad (29)$$

$$\xi = k_1(n+2)\left[\frac{\gamma}{\beta T^2} + \frac{\delta}{(\beta+2n-2)T^{4-2n}} + \frac{\gamma\alpha}{(\beta-n)T^{n+2}} + \frac{k\gamma}{(\beta-2)T^4} + \frac{M}{T^{\beta+2}}\right]^{1/2} \quad (30)$$

Magnitude of the rotation ω is identically zero.

$$\sigma^2 = \frac{(n-1)^2}{3}\left[\frac{\gamma}{\beta T^2} + \frac{\delta}{(\beta+2n-2)T^{4-2n}} + \frac{\gamma\alpha}{(\beta-n)T^{n+2}} + \frac{k\gamma}{(\beta-2)T^4} + \frac{M}{T^{\beta+2}}\right] \quad (31)$$

$$\Lambda = \frac{\alpha}{T^{n+2}} \quad (32)$$

5. CONCLUSION

We have obtained a new class of anisotropic cosmological model with bulk viscous fluid as a source in presence of electromagnetic field.

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The model starts expanding with big-bang at T=0. The expansion decreases as time increases and stops at $n = -2$. Since $\frac{\sigma}{\theta} = \frac{n-1}{\sqrt{3}(n+2)} \neq 0$ for T→∞, therefore the model does not approach isotropy for large value of T, however the model is isotropized when n=1.

The model has a point type singularity for n>0 as T→0, g_{11}→0, g_{22}→0, g_{33}→0. The energy density (ρ) and string tension density (λ) are found to be a decreasing function of time T for $-2 < n < 2$ and approaches to 0 as T→∞. By equation (32), we observe that the cosmological term Λ for the model is also decreasing function of cosmic time T for n>-2.

Hence, in general, the present model represents expanding, shearing and non-rotating universe.

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